Abstract:

In the engineering paper by Groma and Balogh from 1999, a new constitutive relation was derived to describe the density of defects (dislocations) in metal, which is the key ingredient for describing plastic deformation of metals. Currently, this constitutive relation remains the foundation for many more advanced models. However, not all scientists like this constitutive relation, which has resulted in an ongoing debate about its validity.

This debate originates from the phenomenological nature of the derivation of the constitutive relation. The constitutive relation is a 2-species PDE in divergence form for the densities $^{+}$ and , where the ux is given by a non-local and singular interaction term. Groma and Balogh derived it as a many-particle limit of a system of rst order ODEs, which describe the movement of individual defects (dislocations) in the metal. This derivation relies on tools from statistical mechanics, which depends on a phenomenological closure assumption. The choice of this assumption is the reason for the debate.

To solve this debate, we propose a rigorous approach for passing to the many-particle limit. While the singular nature of the interactions between the particles has a regularizing e ect for the continuum densities $^{+}$ and , it's e ect on the system of ODEs gets worse as the number of particles n increases. Therefore, we introduce a regularisation of this singularity, whose length-scale $_n$ converges to 0 as $n \to 1$. Interestingly, the choice of regularization and the convergence speed of $_n \to 0$ are crucial for obtaining the result by Groma and Balogh.