Regularity for Diffuse Re ection Boundary Problem to the Stationary Linearized Boltzmann Equation in a Convex Domain

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Stationary linearized Boltzmann equation in a convex domain in R^3

We consider

$$\begin{cases} r\ f(x; \ ) = L(f); \\ \qquad\qquad x\ 2\ \ ; \\ \qquad\qquad\ 2\ R^3; \end{cases} \qquad (1)$$

where   is a C^2 strictly convex bounded domain in R^3.
Here, we consider hard sphere, cutoff hard potential, or cutoff Maxwellian molecular gases, i.e., 0   1.

Diffuse Re ection Boundary Condition

Diffuse Re ection Boundary Condition:

- Velocity distribution function leaving the boundary is in thermal equilibrium with the boundary temperature.
- There is no net ux on boundary.

Regularity for the time evolutional problem (weakly nonlinear):

$$\frac{\partial}{\partial t} f(x; \ ; t) + \quad \nabla_x f(x; \ ; t) = L(f) + (\ f; f); \tag{2}$$

- Kim (2011 CMP): Discontinuity from boundary in a nonconvex domain.
- Guo, Kim, Tonon, and Trescases (2016 ARMA): BV estimate in a nonconvex domain.
- Guo, Kim, Tonon, and Trescases (2016 Inv. Math.) : Regularity in a convex domain.

All these results are NOT uniform in time.

In (2016 Inv. Math.), they establish weighted C^1 estimate, which grows severely with time. This motivates us to look at the regularity to the stationary solution directly.

Estimates for Kernel

Let $0 < \quad < \quad 1$.

$jk(\; ; \quad)j$

$C_1 j \qquad j^{\;1}(1 + j \; j + j \quad j)^{\;(1 \quad)} e$

Main Theorem

Sketch of proof

Diffuse re ection boundary condition for linearized Boltzmann equation:

Let T(x) be the temperature on the boundary.

For x 2 @ and n(x) < 0;

$$f(x; \ ) = \ (x)M^{\frac{1}{2}} + T(x)(j \ j^2 \ \ \frac{3}{2})M^{\frac{1}{2}}; \qquad (8)$$

where

$$M = M(\ ) = \ \ ^{\frac{3}{2}} e^{j \ \ j^2} :$$

$$(x) = \ \frac{1}{2} T(x) + 2^{p} \overline{\ \ } \int_{n > 0} f(x; \ ) j \ \ n j M^{\frac{1}{2}} d \ : \qquad (9)$$

Let

$$(x) :=$$

- D_f is bounded differentiable provided f is locally Hölder up to boundary.
- We have the desired estimate for rst derivatives of f provided derivatives of , T are bounded and f is locally Hölder.

Transfer regularity from velocity to space

Idea: Combination of averaging or collision and transport can transfer regularity in velocity to space. For time evaluational problem in whole space,

- Velocity averaging lemma (Golse, Perthame, Sentis 1985)
- Mixture lemma (Liu, Yu ARMA 2004)

In present research, we realize this effect for stationary problem in a convex domain by interplaying between velocity and space.

Let $y = x \quad r\hat{}$

$$D_f = 2^{\frac{1}{4}} \int_0^1 \int_0^{\frac{}{2}} \int_0^2 \int_0^{} |x \quad p(x;)|$$

$$e^{\frac{( )}{}r} K(f)(x \quad r\hat{}; \;)|\hat{} \quad n(x)| e^{\frac{2}{2}}\, {}^2 \sin \; drd \; d \; d$$

$$= 2^{\frac{1}{4}} \int_0^1 \int$$

$$e^{\frac{( )}{}|x \quad y|} K(f)(y; \frac{(x \quad y)}{|x \quad y|}) \frac{(x \quad y) \quad n(x)}{|x \quad y|^3} e^{\frac{2}{2}}\, {}^2 dyd$$

$$= 2^{\frac{1}{4}} \int_0^1 \int \int_{R^3}$$

$$e^{\frac{( )}{}|x \quad y|} k(\frac{x \quad y}{|x \quad y|}; \; {}^0) f(y; \; {}^0) \frac{(x \quad y) \quad n(x)}{|x \quad y|^3} e^{\frac{2}{2}}\, {}^2 d \; {}^0 dyd \; : \tag{18}$$

Suppose $g(t)$ is on ∂ passing x and

$$
\begin{aligned}
g(0) &= x; \\
g'(0) &= v;
\end{aligned}
$$

where $v \in T_x\partial$. We define

$$
\nabla^x_v F(x; \ ) = \frac{d}{dt} F(g(t); \ ) \Big|_{t=0} : \qquad (19)
$$

$$
\begin{aligned}
r \; {}_x D_f^{2;} = & \int_0^1 \int_{R^3} \int_{@ \; nB(x;\;)} e^{\frac{(\;)}{}|x \quad y|} k\Big(\frac{x \quad y}{|x \quad y|};\; {}^{0}\Big) \frac{(x \quad y) \quad n(x)}{|x \quad y|^3} \\
& e^{\frac{2}{2}} \; {}^{2} f(x;\; {}^{0})[v \quad n(y)] \, dA(y) \, d \, {}^{0} d \\
& \int_0^1 \int_{R^3} \int_{@ B(x;\;) \backslash} e^{\frac{(\;)}{}|x \quad y|} k\Big(\frac{x \quad y}{|x \quad y|};\; {}^{0}\Big) \frac{(x \quad y)}{|x \quad y|} \quad n(x) \\
& e^{\frac{2}{2}} \; {}^{2} f(x;\; {}^{0})\Big[v \quad \frac{x \quad y}{|x \quad y|}\Big] \frac{1}{2} \, dA(y) \, d \, {}^{0} d \\
=: & \; S \; + B \; :
\end{aligned}
\tag{21}
$$

Thank you!