Abstract:
Mathematical homogenization is an umbrella term for tools and methods used to deduce the macroscopic behavior of a particular medium based on its microscopic properties and usually involves some limit analysis with respect to a length scale parameter. Free boundary problems are usually given via PDEs where the underlying domain (representing some physical medium) is at least partially free to move/deform and where this evolution is not known at the outset. Phase transformation processes (e.g., water/ice or different phases in steel) are typical examples of such problems, where the geometry is allowed to evolve and where microscopic effects (growing of nucleation cells) determine the macroscopic properties of the system.
In this talk, we will first review some of the usual approaches in the homogenization of phase transformation problems like phase field models, dimensional reduction, and the method of formal asymptotics and discuss their advantages and shortcomings. Then, we will present some of our exploratory work on the rigorous homogenization of free boundary problems and, in particular, discuss the emerging difficulties and open issues like regularity results and uniform estimates.