Abstract:
We study the existence of unique solutions for nonlinear PDEs in the realm of the Vlasov-Poisson-Fokker-Planck (VPFP) systems. We introduce a unifying kinetic model preserving mass, moment and energy. Then, we construct and analyze i) a finite volume scheme for the three dimensional VPFP, ii) a fully discrete scheme based on a backward-Euler (BE) approximation in time combined with a mixed finite element method for a discretization of the Poisson equation in the spatial domain and a streamline-diffusion (SD) finite element approximation in the phase-space variables for the Vlasov equation, and iii) an abstract adaptivity procedure based on the *hp* finite element for the VPFP, where *h* is the mesh parameter and *p* the spectral order of the approximation.
The Fokker-Planck term is treated as a diffusion transport equation using the characteristic Galerkin approach. We prove the stability estimates and derive the optimal convergence rates depending upon the compatibility of the finite element meshes, used for the discretizations of the spatial variable in Poisson (mixed) and Vlasov (SD/DG) equations, respectively. The error estimates for the Poisson equation are based on using Brezzi-Douglas-Marini (BDM) elements in $_2$ and $^-$, $s > 0$, norm0 Td(3T.0 Td[(>)(3W)Fi)(7e)(3(Lo)rm0 Td(3T.07(7)Tj-po)(2