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BOUNDARY LAYERS AND SHOCK PROFILES FOR THE DISCRETE BOLTZMANN EQUATION FOR MIXTURES

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gases A and B reads

$$\begin{cases} \dfrac{\partial f_i^A}{\partial t} + \xi_i^A \cdot \nabla_{\mathbf{x}} f_i^A = Q_i^{AA}(f^A, f^A) + Q_i^{BA}(f^B, f^A), \ i = 1, \ldots, n_A, \\ \dfrac{\partial f_j^B}{\partial t} + \xi_j^B \cdot \nabla_{\mathbf{x}} f_j^B = Q_j^{AB}(f^A, f^B) + Q_j^{BB}(f^B, f^B), \ j = 1, \ldots, n_B \end{cases} \tag{1}$$

where $V^\alpha = \{\xi_1^\alpha, \ldots, \xi_{n_\alpha}^\alpha\} \subset \mathbb{R}^d$, $\alpha \in \{A, B\}$ are finite sets of velocities, $f_i^\alpha = f_i^\alpha(\mathbf{x}, t) = f^\alpha(\mathbf{x}, t, \xi_i^\alpha)$ for $i = 1, \ldots, n_\alpha$, and f^α

and by the relations (2), with $h = \left(h^A, h^B\right)$,

$$
\begin{aligned}
&\langle h, Q(f,g)\rangle = \\
&= \frac{1}{8}\sum_{i,j,k,l=1}^{n_A} \Gamma_{ij}^{kl}(A,A)\left(h_i^A + h_j^A - h_k^A - h_l^A\right)\left(f_k^A g_l^A + g_k^A f_l^A - f_i^A g_j^A - g_i^A f_j^A\right) \\
&+ \frac{1}{4}\sum_{i,k=1}^{n_A}\sum_{j,l=1}^{n_B} \Gamma_{ij}^{kl}(B,A)\left(h_i^A + h_j^B - h_k^A - h_l^B\right)\left(f_k^A g_l^B + g_k^A f_l^B - f_i^A g_j^B - g_i^A f_j^B\right) \\
&+ \frac{1}{8}\sum_{i,j,k,l=1}^{n_B} \Gamma_{ij}^{kl}(B,B)\left(h_i^B + h_j^B - h_k^B - h_l^B\right)\left(f_k^B g_l^B + g_k^B f_l^B - f_i^B g_j^B - g_i^B f_j^B\right).
\end{aligned} \tag{3}
$$

A vector $\phi = \left(\phi^A, \phi^B\right)$ is a collision invariant if and only if

$$
\phi_i + \phi_j = \phi_k + \phi_l, \tag{4}
$$

for all indices $1 \leq i,k \leq n_\alpha$, $1 \leq j,l \leq n_\beta$ and $\alpha, \beta \in \{A, B\}$, such that $\Gamma_{ij}^{kl}(\alpha,\beta) \neq 0$. By the relation (3)

$$
\begin{aligned}
\langle \phi, Q(f,f)\rangle = {} & \frac{1}{4}\sum_{i,j,k,l=1}^{n_A} \Gamma_{ij}^{kl}(A,A)\left(\phi_i^A + \phi_j^A - \phi_k^A - \phi_l^A\right)\left(f_k^A f_l^A - f_i^A f_j^A\right) \\
&+ \frac{1}{2}\sum_{i,k=1}^{n_A}\sum_{j,l=1}^{n_B} \Gamma_{ij}^{kl}(B,A)\left(\phi_i^A + \phi_j^B - \phi_k^A - \phi_l^B\right)\left(f_k^A f_l^B - f_i^A f_j^B\right) \\
&+ \frac{1}{4}\sum_{i,j,k,l=1}^{n_B} \Gamma_{ij}^{kl}(B,B)\left(\phi_i^B + \phi_j^B - \phi_k^B - \phi_l^B\right)\left(f_k^B f_l^B - f_i^B f_j^B\right).
\end{aligned} \tag{5}
$$

which is zero, independently of our choice of non-negative vector f ($f_i \geq 0$ for all $1 \leq i \leq n$), if and only if ϕ is a collision invariant.

We consider below (even if this restriction is not necessary in our general reasoning) only DVMs, such that any collision invariant is of the form

$$
\phi = \left(\phi^A, \phi^B\right), \text{ with } \phi^\alpha = \phi^\alpha(\boldsymbol{\xi}) = a_\alpha + m_\alpha \mathbf{b}\cdot\boldsymbol{\xi}_\alpha + cm_\alpha\left|\boldsymbol{\xi}_\alpha\right|^2, \tag{6}
$$

for some constant $a_A, a_B, c \in \mathbb{R}$ and $\mathbf{b} \in \mathbb{R}^d$. In this case the equation

$$
\langle \phi, Q(f,f)\rangle = 0
$$

has the general solution (6). Discussions on constructions of DVMs for binary mixtures can be found in e.g. [11],[12],[20],[21],[14] and [15].

A binary Maxwellian distribution (or just a bi-Maxwellian) is a function $M = \left(M^A, M^B\right)$, such that

$$
Q(M,M) = 0 \text{ and } M_i \geq 0 \text{ for all } 1 \leq i \leq n.
$$

All bi-Maxwellians are of the form

$$
M = e^{\phi}, \text{ i.e. } M = \left(M^A, M^B\right), \text{ with } M^\alpha = e^{\phi^\alpha} = e^{a_\alpha + m_\alpha \mathbf{b}\cdot\boldsymbol{\xi}_\alpha + cm_\alpha\left|\boldsymbol{\xi}_\alpha\right|^2}, \tag{7}
$$

where $\phi = \left(\phi^A, \phi^B\right)$ is given by Eq.(6). Assuming that f is non-negative, we let

$\varphi = \log f$ in Eq.(5) and obtain that

$$\langle \log f, Q(f,f)\rangle = \frac{1}{4}\sum_{i,j,k,l=1}^{n_A}\Gamma_{ij}^{kl}(A,A)\left(f_k^A f_l^A - f_i^A f_j^A\right)\log\frac{f_i^A f_j^A}{f_k^A f_l^A}$$
$$+\frac{1}{2}\sum_{i,k=1}^{n_A}\sum_{j,l=1}^{n_B}\Gamma_{ij}^{kl}(B,A)\left(f_k^A f_l^B - f_i^A f_j^B\right)\log\frac{f_i^A f_j^B}{f_k^A f_l^B}$$
$$+\frac{1}{4}\sum_{i,j,k,l=1}^{n_B}\Gamma_{ij}^{kl}(B,B)\left(f_k^B f_l^B - f_i^B f_j^B\right)\log\frac{f_i^B f_j^B}{f_k^B f_l^B} \leq 0,$$

with equality if and only if

$$f_k^\alpha f_l^\beta = f_i^\alpha f_j^\beta$$

for all indices $1 \leq i,k \leq n_\alpha$, $1 \leq j,l \leq n_\beta$ and $\alpha,\beta \in \{A,B\}$, such that $\Gamma_{ij}^{kl}(\alpha,\beta) \neq 0$, or equivalently, if and only if f is a bi-Maxwellian. Hence, f is a bi-Maxwellian if and only if $\log f$ is a collision invariant.

For a bi-Maxwellian $M = \left(M^A, M^B\right)$, we obtain, by denoting

$$f = M + \sqrt{M}h, \tag{8}$$

in Eq.(1), the system

$$\frac{\partial h}{\partial t} + (\boldsymbol{\xi}\cdot\nabla_{\mathbf{x}})h = -Lh + S(h),$$

where $(\boldsymbol{\xi}\cdot\nabla_{\mathbf{x}})h = (\boldsymbol{\xi}_1^A\cdot\nabla_{\mathbf{x}}h_1^A, \dots, \boldsymbol{\xi}_{n_A}^A\cdot\nabla_{\mathbf{x}}h_{n_A}^A, \boldsymbol{\xi}_1^B\cdot\nabla_{\mathbf{x}}h_1^B, \dots, \boldsymbol{\xi}_{n_B}^B\cdot\nabla_{\mathbf{x}}h_{n_B}^B)$. Furthermore, L is the linearized collision operator ($n \times n$ matrix, with $n = n_A + n_B$) given by

$$Lh = -\frac{2}{\sqrt{M}}Q(M, \sqrt{M}h), \tag{9}$$

and the quadratic part S is given by

$$S(h,h) = \frac{1}{\sqrt{M}}Q(\sqrt{M}h, \sqrt{M}h). \tag{10}$$

By Eq.(3) and the relations

Hence, the matrix L is symmetric, i.e.

$$\langle g, Lh\rangle = \langle Lg, h\rangle$$

for all g and h, and semi-positive, i.e.

$$\langle h, Lh\rangle \geq 0$$

for all h. Also $\langle h, Lh\rangle = 0$ if and only if

$$\sqrt{M_k}\, h_l + \sqrt{M_l}\, h_k = \sqrt{M_i}\, h_j + \sqrt{M_j}\, h_i \tag{11}$$

for all indices $1 \leq i, k \leq n_\alpha$, $1 \leq j, l \leq n_\beta$, and $\alpha, \beta \in \{A, B\}$, satisfying $\sigma_{ij}^{kl}(\alpha, \beta) \neq 0$. We let $h = \phi\sqrt{M}$ in Eq.(11), and obtain Eq.(4), by the relations $M_i M_j = M_k M_l \neq 0$. Hence,

$$Lh = 0 \text{ if and only if } h = \phi\sqrt{M},$$

where ϕ is a collision invariant. In consequence,

$$\left\langle S(h, h), \phi\sqrt{M}\right\rangle = \langle Q(f, f), \phi\rangle + \left\langle h, L\phi\sqrt{M}\right\rangle = 0$$

for all collision invariants ϕ.

In the planar stationary case our system for mixtures reads

$$D\frac{dh}{dx} + Lh = S(h, h),\ x \in \mathbb{R},$$

where $D = \begin{pmatrix} D_A & 0 \\ 0 & D_B \end{pmatrix}$, with $D_\alpha = \mathrm{diag}(\xi_1^{\alpha,1}, \ldots, \xi_{n_\alpha}^{\alpha,1})$, and the operators L and S are given by Eqs.(9)-(10).

We consider below the case when D is non-singular, i.e. when all $\xi_i^{\alpha,1} \neq 0$ are non-zero. For the case of singular matrices D, see Remark 5 below.

We denote by $n^\pm$, where $n^+ + n^- = n$, and $m^\pm$, with $m^+ + m^- = q$, the numbers of positive and negative eigenvalues (counted with multiplicity) of the matrices D and $D^{-1}L$ respectively, and by m^0 the number of zero eigenvalues of $D^{-1}L$. Moreover, we denote by k^+, k^-, and l, with $k^+ + k^- = k$, where $k + l = p$, the numbers of

We also define the projections $R_+ : \mathbb{R}^n \to \mathbb{R}^{n^+}$ and $R_- : \mathbb{R}^n \to \mathbb{R}^{n^-}$, $n^- = n - n^+$, by

$$R_+ s = s^+ = s_1, \dots, s_{n_A^+}, s_{n_A+1}, \dots, s_{n_A+n_B^+}$$

(iv) Assume that the condition (19) *is ful lled. Then the system* (12) *with the boundary conditions (Q) and* (16) *has a unique solution with the asymptotic ow* (21) *if the* $k + l$ *parameters* $_{k^+ +1}; :::; \, _k$ *and* $\#_1; :::; \#_l$, $\#_i = \, _i + \, _i$, *are prescribed.*

Especially, for the homogeneous system (12) with $g = 0$, the condition (20) is reduced to

$$h_0 \; 2 \; CU_+ .$$

Lemma 3.2. *Let* $D^+ = \begin{pmatrix} D_A^+ & 0 \\ 0 & D_B^+ \end{pmatrix}$ *and* $D = \begin{pmatrix} D_A & 0 \\ 0 & D_B \end{pmatrix}$, *cf. Eq.*(15). *Then*

i) *the condition* (18) *is ful lled, if*

$$C^T D^+ C < D \quad \text{on } R \quad X_+ ;$$

ii) *the conditions* (17) *and* (19) *are ful lled, if*

$$C^T D^+ C \quad D \quad \text{on } R \;\; U_+ \text{ and } R \;\; X_+ , \text{ respectively.}$$

Corollary 1. *If* $C = 0$, *then the conditions* (17)-(19) *are ful lled.*

In particular, $u_1^+; :::; u_{m^+}^+; y_1^+; :::; y_{k^+}^+; z_1^+; :::; z_l^+$ *is a basis of* R^{n^+}.

We consider the non-linear system

$$D \, \frac{df}{}$$

Theorem 3.4. *Let condition* (17) *be fulfilled and assume that*

$$h_0, Ce^{xB^{-1}L}D^{-1}S(f(x), f(x)) \in CU_+ \text{ for all } x \in \mathbb{R}_+,$$
$$\text{with } U_+ = \mathrm{span}(u : Lu = \lambda Du, \ \lambda > 0) = \mathrm{span}\,(u_1, \dots, u_{m^+}) .$$

Then there is a positive number δ_0*, such that if*

$$|h_0| \leq \delta_0,$$

then the system (22) *with the boundary conditions (O),*(16)*, has a locally unique solution (with respect to the norm* $|\cdot|$ *).*

Remark 3. If condition (17) is fulfilled, then the condition

$$h_0 \in CU_+$$

implies that we have $k^+ + l$ conditions on h_0.

Remark 4. If the conditions

$$CU_- \subseteq CU_+, \tag{24}$$

$$\begin{aligned} \text{with } U_- &= \mathrm{span}\,(\{u \mid Lu = \lambda Du, \text{ with } \lambda < 0\} \cup \{z_1, \dots, z_l\}) \\ &= \mathrm{span}\,(u_{m^++1}, \dots, u_q, z_1, \dots, z_l)\,; \end{aligned}$$

and (13) are fulfilled, then

$$Ce^{xD^{-1}L}D^{-1}S(f, f) \in CU_+ \text{ for all } x \in \mathbb{R}_+.$$

Remark 5. All our results for half-space problems can be extended in a natural way, to yield also for singular matrices D, if

$$N(L) \cap N(D) = \{0\}.$$

4. Applications to shock profiles.

We are interested in solutions to the problem

$$(D - cI)\frac{dF_i}{dy} = Q_i(F, F)\,,\ i = 1, \dots, n,\ c \in \mathbb{R}, \tag{25}$$

such that

$$F \to M_\pm \text{ as } y \to \pm\infty,$$

where $M_\pm$ are two bi-Maxwellians and $D = \begin{pmatrix} D_A & 0 \\ 0 & D_B \end{pmatrix}$, with D_α from Eqs.(14)-(15). Here $F = (F_1, \dots, F_n)$, with $F_i = F_i(y) = F(y, \xi_i)$, $i = 1, \dots, n$.

Note that shifting the velocity variable in the continuous Boltzmann equation doesn't change the velocity set, while for a finite set of velocities a shift in the velocity variable changes the set of velocities. However, if we want to end up with a specific set of velocities after a given shift in the velocity variable, we can always start with a suitably shifted set of velocities. Note also that changing c in the discrete case can change the number of positive (and negative) eigenvalues of the matrix $D - cI$, and thereby the number of positive (and negative) eigenvalues of the matrix $(D - cI)^{-1}L$ can change also away from the degenerate values of c.

We denote by $\{\phi_1, \dots, \phi_p\}$ ($p = d + 3$ for normal DVMs for binary mixtures) a basis for the vector space of collision invariants. If we multiply Eq.(25) scalarly by ϕ_i, $1 \leq i \leq p$, and integrate over $\mathbb{R}$, then we obtain that the bi-Maxwellians M_- and M_+ must fulfill the Rankine-Hugoniot conditions

$$\langle M_+, \phi_i \rangle_{D-cI} = \langle M_-, \phi_i \rangle_{D-cI}\,,\ i = 1, \dots, p.$$

We make the following assumptions on our DVMs.

1. There is a number c_0 ("speed of sound"), with the following properties:
   [i] rank$(K) = p$ 1, where K is the p p matrix with the elements

$$k_{ij} = hM_{+} \ {}_{i}; \ {}_{j}i_{D} \ c_0 I \ .$$

The rank of K is independent of the choice of the basis f ${}_1; :::;$ ${}_p g$. In other words, there is a unique (up to its sign) vector

Then,

$$K = \begin{pmatrix} c\,{}^{A}_{1} & 0 & {}^{A}_{2} & c\,{}^{A}_{3} & & & \\ 0 & c\,{}^{B}_{1} & {}^{B}_{2} & c\,{}^{B}_{3} & & & \\ {}^{A}_{2} & {}^{B}_{2} & c\,{}_{2} & {}_{4} & & & \\ c\,{}^{A}_{3} & c\,{}^{B}_{3} & {}_{4} & c\,{}_{5} & & & \\ & & & & c\,{}_{6} & & \\ & & & & & \ddots & \\ & & & & & & c\,{}_{d+4} \end{pmatrix};$$

where $K = (h\ {}_{i+1};\ {}_{j+1}i_D\ {}_{cI})$, ${}^{A}_{1} = h\ {}_{0};\ {}_{0}i$, ${}^{A}_{2} = h\ {}_{0};\ {}_{2}i_D$, ${}^{A}_{3} = h\ {}_{0};\ {}_{3}i$, ${}^{B}_{1} = h\ {}_{1};\ {}_{1}i$, ${}^{B}_{2} = h\ {}_{1};\ {}_{2}i_D$, ${}^{B}_{3} = h\ {}_{1};\ {}_{3}i$, ${}_{2} = {}^{A}_{2} + {}^{B}_{2} = h\ {}_{2};\ {}_{2}i$, ${}_{4} = h\ {}_{2};\ {}_{3}i_D$ and ${}_{i+2} = h\ {}_{i};\ {}_{i}i$, $i = 3;:::;d + 2$. Hence,

$$\begin{aligned} &\det(K) \\ &= c^{d}\ {}_{6}:::\ {}_{d+4}(c^{2}(\ {}^{A}_{1}\ {}^{B}_{1}\ {}_{2}\ {}_{5}\quad {}^{A}_{1}\ {}_{2}(\ {}^{B}_{3})^{2}\quad {}^{B}_{1}\ {}_{2}(\ {}^{A}_{3})^{2}) + (\ {}^{A}_{2}\ {}^{B}_{3}\quad {}^{B}_{2}\ {}^{A}_{3})^{2} \\ &+ 2\ {}_{4}(\ {}^{A}_{1}\ {}^{B}_{2}\ {}^{B}_{3} + {}^{B}_{1}\ {}^{A}_{2}\ {}^{A}_{3})\quad (\ {}^{A}_{1}(\ {}^{B}_{2})^{2} + {}^{B}_{1}(\ {}^{A}_{2})^{2})\ {}_{5}\quad {}^{A}_{1}\ {}^{B}_{1}\ {}^{2}_{4}), \end{aligned}$$

and the degenerate values of c (the values of c for which l 1) are

$$c_0 = 0 \text{ and } c\ = \sqrt{\frac{X}{{}_{2}(\ {}^{A}_{1}\ {}^{B}_{1}\ {}_{5}\quad {}^{A}_{1}(\ {}^{B}_{3})^{2}\quad {}^{B}_{1}(\ {}^{A}_{3})^{2})}},\ \text{where}$$

$$X = {}^{A}_{1}\ {}^{B}_{1}\ {}^{2}_{4} + (\ {}^{A}_{1}$$

1 1(A(2 B 1 (A

with

$$L = s\begin{pmatrix} l_1 & l_1 & _1\sqrt{b} & _1\sqrt{b} & _2\sqrt{d} & _2\sqrt{d} \\ l_1 & l_1 & _1\sqrt{b} & _1\sqrt{b} & _2\sqrt{d} & _2\sqrt{d} \\ _1\sqrt{b} & _1\sqrt{b} & l_2 & l_2 & _2\sqrt{bd} & _2\sqrt{bd} \\ _1\sqrt{b} & _1\sqrt{b} & l_2 & l_2 & _2\sqrt{bd} & _2\sqrt{bd} \\ _2\sqrt{d} & _2\sqrt{d} & _2\sqrt{bd} & _2\sqrt{bd} & _2(1+b) & _2(1+b) \\ _2\sqrt{d} & _2\sqrt{d} & _2\sqrt{bd} & _2\sqrt{bd} & _2(1+b) & _2(1+b) \end{pmatrix};$$

where $l_1 = \ _1b + \ _2d$ and $l_2 = \ _1 + \ _2d$, and

$$S(h;h) = \sqrt{s}(\ _1q_1(\sqrt{b};\ \sqrt{b};\ 1;1;0;0) + \ _2q_2(\sqrt{d};\ \sqrt{d};0;0;\ 1;1) + \ _2q_3(0;0;\sqrt{d};\ \sqrt{d};\ \sqrt{b};\sqrt{b})).$$

The linearized operator L is symmetric and semi-positive and has the null-space

$$N(L) = \mathrm{span}(y_1; y_2; y_3; y_4), \text{ with}$$

$$y_1 = (1;1;0;0;0;0); y_2 = (0;0;1;1;0;0);$$
$$y_3 = (0;0;0;0;1;1); \text{ and } y_4 = (1;\ 1;\sqrt{b};\ \sqrt{b};\sqrt{d};\ \sqrt{d})$$

where we for $c \neq 0$ can replace y_4 with

$$y_4 = \ 1 + c; 1\ \ c; \sqrt{b}(1+c); \sqrt{b}(1\ \ c); \sqrt{d}(m+c); \sqrt{d}(m\ \ c)\ :$$

Then we obtain

$$K = \begin{pmatrix} 2c & 0 & 0 & 0 \\ 0 & 2c & 0 & 0 \\ 0 & 0 & 2c & 0 \\ 0 & 0 & 0 & 2c \end{pmatrix}$$

8 9.9626 Tf 6.846 0 Td [(()]TJ/F11 9.9626 Tf 3.874 0 Td [(m)]TJ/F8 .96

with the corresponding eigenvectors

$$\begin{cases} u_1 = \left(\frac{\sqrt{b}}{1 \quad c}; \frac{\sqrt{b}}{1+c}; \frac{1}{\sqrt{1 \quad c}}; \frac{1}{\sqrt{1+c}}; 0; 0\right) \\ u_2 = \left(\frac{\sqrt{d}}{1 \quad c}; \frac{\sqrt{d}}{1+c}; \frac{\sqrt{bd}}{1 \quad c}; \frac{\sqrt{bd}}{1+c}; \frac{1+b}{m \quad c}; \frac{1+b}{m+c}\right) \end{cases} .$$

Plugging $h = \quad u_1 + \quad u_2$ in Eq.(28) and multiplying scalarly by $(D \quad cI)\,u_i$, we obtain the two equations

$$\begin{cases} \frac{d}{dy} + \quad {}_1 \quad = k \\ \frac{d}{dy} + \quad {}_2 \quad = k \quad {}^2 \end{cases} ,$$

with $k = \frac{2 \quad {}_2 \sqrt{sdc}\,(1+b)\,(m \quad 1)}{(1 \quad c^2)\,(m^2 \quad c^2)}$. The solutions are

$$\begin{cases} \quad = \frac{\quad {}_2}{k + C_1 e^{\quad {}_2 y}} \\ \quad = \frac{C_2 e^{\quad {}_1 y}}{k e^{\quad {}_2 y} + C_1} = \frac{C_2 e^{(\quad {}_2 \quad {}_1) y}}{k + C_1 e^{\quad {}_2 y}} \end{cases} \quad \text{or} \quad \begin{cases} \quad = 0 \\ \quad = C_2 e^{\quad {}_1 y} \end{cases} .$$

The parameter $C_1 \neq 0$ reflects the invariance of our equation under shifts in the invariant variable y. The sign of C_1 is, however, defined uniquely. It must be the same as the sign of k.

Let $c_+ < c < m$ or $c \quad < c < \frac{1+b+dm}{1+b+d}$. If $c_+ < c < m$, then $\quad {}_2 > 0$ and $\quad {}_1 < 0$, and hence, $\lim_{y \to \infty} \quad = 0$ implies that C_2

with $\frac{1 + b + dm^2}{1 + b + d} < c < c_+$ or $m < c < c$; and in a similar way as above, we obtain

$$f(y) = M \quad + \frac{2}{k + C_1 e^{\ 2y}} M^{1=2} u_2$$

and

$$M_+ = M \quad + \frac{2}{k} M^{1=2} u_2 .$$

7. Boundary layers for the 6+4-velocity model with a moving wall. Let c be a real number such that c 2 f $m;$ $1; 0; 1; mg$. We assume that $m > 1$. The cases $m < 1$ and $m = 1$ can be studied in a similar way. We de ne the projections $R_+ : \mathbb{R}^6$! $\mathbb{R}^{n^+}$, $n^+ = n_A^+ + n_B^+$, and R : $\mathbb{R}^6$! $\mathbb{R}^n$, $n = 6 \quad n^+$, by

$$R_+ s = s^+ \text{ and } R \ s = s \ , \text{ where}$$

$$s^+ = \begin{cases} s_5 \text{ if } 1 < c < m \\ (s_1; s_3; s_5) \text{ if } 1 < c < 1 \end{cases}$$

u(

s = s ,87 0 Td [(=)]TJ/F11 9.9626 Tf 10.516 0 Td [(s)]TJ/F13 6.97389Tf 4.67 4.114 Td [( )]T:5
s =s =.9738Tf7.564-1.49499Td[(;)->

If $h_0 = 0$, then we always have the trivial solution $f = 0$; and if 1

We see that the number of conditions on h_0, in fact, equals $k^+ + l$, where k^+ and l can be found in table 29.

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