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the discrete Boltzmann equation .r4zmann equationplace at the
the non-condensable gas is diffusively re ected at the condensed
are distributed according to a given distribution. The conditions,
the existence of a unique solution of the problem are investigated
is suf ciently close to the Maxwellian at in nity and that the total m
solutions and solvability conditions are found for a speci c simpli

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INTRODUCTION

Half-space problems for the Boltzmann equation are important in the study of the asymptotic behavior of the solutions of boundary value problems of the Boltzmann equation for small Knudsen numbers [1, 2]. For single-component gases half-space problems are well-studied mathematically both for the continuous Boltzmann equation as well as the discrete Boltzmann equation, see [3, 4, 5] and references therein. In the present paper we present some of ou results for the discrete Boltzmann equation for binary mixtures, recently obtained in [6] and [7]. We do consider the case of a binary mixture of two vapors, but our main objective is the case of a condensing vapor in the presence of a non-condensable gas, cf. [8] , for which the main result is presented in Theorem 2. In the latter case we also present explicit solutions and solvability conditions for a reduced 6+4-velocity model in the case of a ow symmetric around the x-axis [7]. We start by reviewing some general properties for the planar stationary discrete Boltzmann equation for binary mixtures.

The planar stationary discrete Boltzmann equation for a binary mixture of the gases A and B reads [6]

$$\begin{cases} \xi_i^{A,1}\dfrac{dF_i^A}{dx} = Q_i^{AA}(F^A,F^A)+Q_i^{BA}(F^B,F^A),\ i=1,\dots,n_A, \\ \xi_j^{B,1}\dfrac{dF_j^B}{dx} = Q_j^{AB}(F^A,F^B)+Q_j^{BB}(F^B,F^B),\ j=1,\dots,n_B, \end{cases} \qquad (1)$$

where $\mathcal{V}_a = \{\xi_1^a,\dots,\xi_{n_a}^a\} \subset \mathbb{R}^d$, $a,b \in \{A,B\}$, are nite sets of velocities, $F_i^a = F_i^a(x) = F^a(x,\xi_i^a)$ for $i=1,\dots,n_a$, and $F^a = F^a(x,\xi)$ represents the microscopic density of particles (of the gas a) with velocity ξ at position $x \in \mathbb{R}$.

We denote by m_a the mass of a molecule of gas a. Here and below $a, b \in \{A, B\}$.

For a function $g^a = g^a(x)$ (possibly depending on more variables than x), we will identify g^a with its restrictions to the set V^a.

BINARY MIXTURES OF TWO VAPORS

In this section we consider the case of a binary mixture of two vapors [6] (and as a particular case the case of a single vapor [5]), to give the possibility to compare with the results for the case of a condensing vapor with a non-condensable gas present [7], presented in the next section. We assume that our DVMs are normal considered as binary mixtures. It is also preferable that the DVMs for the gases A and B are normal, even if this doesn't affect our results.

For a bi-Maxwellian $M = (M^A; M^B)$, we obtain, by substituting $F = M + \sqrt{M} f$ in Eq. (4), the system

$$D\frac{df}{dx} + Lf = S(f;f), \tag{5}$$

where the linearized operator L is a symmetric and semi-positive matrix, with the null-space

$$\begin{aligned} N(L) &= \mathrm{span}(R_A M^{1/2}; R_B M^{1/2}; M^{1/2}x^1; \ldots; M^{1/2}x^d; M^{1/2}|x|^2), \text{ where} \\ R_A h &= (h_1; \ldots; h_{n_A}; 0; \ldots; 0) \text{ and } R_B h = (1 - R_A)h \text{ if } h \in \mathbb{R}^n, \text{ with } n = n_A + n_B, \end{aligned}$$

and the quadratic part $S(f;f)$ belong to the orthogonal complement of $N(L)$ [6].

At the far end we assume that

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CONDENSING VAPOR FLOW IN THE PRESENCE OF A NON-CONDENSABLE GAS

In this section we study distributions F, such that $F \to M^{A}; 0$

For a condensing vapor flow (i.e. with $b < 0$, where b is the first component of $\mathbf{b}$ in Eq.(2)), we have $k_B \leq 1$. Moreover, under condition (11), $k_B = 1$ and $k_B^+ = l_B = 0$. However, it is enough for us that $k_B^+ = l_B = 0$, i.e. that

$$k_B = p_B. \tag{14}$$

Conjecture 1 For a normal DVM (for gas A) fulfilling the symmetry relations (8) there is a critical number $b_+ > 0$, such that

	$b < -b_+$	$b = -b_+$	$-b_+ < b < 0$	$b = 0$	$0 < b < b_+$	$b = b_+$	$b_+ < b$
k_A^+	0	0	1	1	$d+1$	$d+1$	$d+2$
l_A	0	1	0	d	0	1	0

(15)

Conjecture 1 is true for the continuous Boltzmann equation [12], where b_+ is the speed of sound. We assume that we have a DVM that restricted to gas A fulfills Conjecture 1, at least in the case of condensation, i.e. for $b < 0$. The number b_+ has been calculated for a plane axially symmetric 12-velocity model (assuming that the solution is symmetric with respect to the x-axis) in [7].

By condition (14), $\dim(\operatorname{span}\{u : L_{AB}u = \lambda D_B u, \lambda > 0\}) = n_B^+$, see [13, 11, 7]. We assume that

$$\dim \operatorname{span}\left(U_B^+\right) = n_B^+ - 1, \text{ where } U_B^+ = \left(R_+^B - CR_-^B\right)\left\{u : L_{AB}u = \lambda D_B u, \lambda > 0\right\}, \tag{16}$$

but, also that

$$\dim \operatorname{span}\left(\tilde{U}_B^+\right) = n_B^+, \text{ where } \tilde{U}_B^+ = U_B^+ \cup \left(R_+^B - CR_-^B\right)\left\{\sqrt{M^B}\right\}. \tag{17}$$

If we would have had $\dim \operatorname{span}\left(U_B^+\right) = n_B^+$, then $f^B(x) = 0$, i.e. the non-condensable gas would have been absent.

For $-b_+ < b < 0$ we will also assume that

$$R_+^A\sqrt{M^A} \notin R_+^A \operatorname{span} U_A^+, \text{ with } U_A^+ = \{u : L_{AA}u = \lambda D_A u, \lambda > 0\}; \tag{18}$$

or, equivalently, since $\dim(R_+^A \operatorname{span} U_A^+) = n_A^+ - 1$ by Eq.(15) [13, 11, 7],

$$\dim(R_+^A \operatorname{span} \tilde{U}_A^+) = n_A^+, \text{ with } \tilde{U}_A^+ = U_A^+ \cup \left\{\sqrt{M^A}\right\}.$$

In fact, we can replace $\sqrt{M^A}$ in assumption (18) by any possible vector $y \in N(L_{AA})$, such that

$$\left(L_{BA} f^B, y\right) = \left(S_{BA}(f^B, f^A), y\right) = 0.$$

We fix $e = \min\{|h_0|, 1\}$ and the total mass of the gas B to be m_B^{tot}, i.e.

$$e m_B \sum_{i=1}^{n_B} \int_0^\infty \sqrt{M^B} f_i^B(x)\, dx = m_B^{tot}, \tag{19}$$

for a given positive constant m_B^{tot}. The case $m_B^{tot} = 0$, corresponds to the case of single 1 Td-(v)15(en)-s96267626 Tf 6.2d [(+

A REDUCED 6+4 - VELOCITY MODEL

In this section we present an exact solution and solvability condition (see [7] for a complete presentation) when the vapor, gas A, is modeled by a six-velocity model with velocities

$$x_1^A = (1;0)$$

and

$$l^{B} = \frac{s^{A}}{m}(s_2 + s_3 q)(p \quad 1) > 0 \text{ and } u^{B} = (\sqrt{p}; 1);$$

respectively.

The new boundary conditions become

$$f_1^A(0); f_2^A(0) = \frac{1}{\sqrt{s^A q}} \sqrt{q}(s_0^A \quad s^A); s_0^A q_0 \quad s^A q \quad \text{and } f_1^B(0) = \sqrt{p}\, f_2^B(0) \tag{22}$$

at the condensed phase, and at the far end

$$f^A \to 0 \text{ and } f^B \to 0 \text{ as } x \to \infty. \tag{23}$$

We decompose

$$f^A = m_1^A y_1^A + m_2^A y_2^A + m_3^A \infty$$

total amount of the gas B, or on the closeness of the far Maxwellian and the Maxwellian at the wall for the gas A to obtain a solution. However, smallness assumptions might be needed to obtain positivity of the solution.

For the case of a condensing vapor ow (symmetric around the x